GROWTH OF SMALL PERTURBATIONS IN A BOUNDARY LAYER ON AN ELASTIC SURFACE

V. B. Amfilokhiev, V. V. Droblenkov, and A. S. Zavorokhina

UDC 532.526

We consider the behavior of small, growing perturbations in a laminar boundary layer on an elastic surface.

On the basis of the analysis we assume the usual Orr–Sommerfeld equation with boundary conditions for the surface, investigating under the action of a perturbing pressure p only small normal deformations $y\delta = kp \exp(i\theta)$ [1]:

$$f(y)|_{y\to\infty} < M < \infty, \quad \alpha f(1) + f'(1) = 0, \quad f'(0) = 0 \tag{1}$$

$$-\frac{kc}{i\alpha R} f'''(0) + f(0)\left[\exp(-i\theta) - kcu'(0)\right] = 0$$

Here $f = f(y)$ is the dimensionless amplitude of the stream function of the perturbing motion, y is the dimensionless transverse coordinate (y = 1 on the outer boundary of the boundary layer of thickness δ, and y = 0 on a streamlined surface in the equilibrium configuration), α is the wave number, u = u(y) is the ratio of the longitudinal component of the velocity of the principal motion in the boundary layer to the velocity U on its outer boundary, $c = c_r + ic_i$ is a complex quantity containing the ratio of the phase velocity of propagation of the perturbation wave c_B to U ($c_r = c_B/U = \beta_r\delta/\alpha U$) and the dimensionless coefficient of growth of perturbations with time ($c_i = \beta_i\delta/\alpha U$), β_r is the circular frequency of the perturbation wave, β_i is the coefficient of growth of perturbations with time, $R = U\delta/\nu$ is the local Reynolds number, ν is the kinematic coefficient of viscosity of the liquid, k is the compliance coefficient of the surface, θ is the angle of phase shift between the deformation of the surface and the perturbing pressure.

If we use the usual form for the particular solutions of the Orr–Sommerfeld equation [2], then from the boundary conditions (1) we can obtain an approximate characteristic equation, in which terms of order $(\alpha R)^{-1}$ and higher are dropped:

$$F = \frac{c}{u_0' y_k}\left(\frac{z}{1+z} + \frac{kcu_0'}{\exp(-i\theta) - kcu_0'}\right) \tag{2}$$

where the index k corresponds to the critical layer ($y = y_k$ for $u = c_r$) and the index 0 corresponds to the coordinate y = 0.

Equation (2) outwardly does not differ from the equation for neutral oscillations [1], but the quantities that appear in it must be calculated taking account of $c_i = 0$. The quantity z, which depends on the velocity profile of the principal flow, is not related to the characteristics of the streamlined surface and has the same form as for the case of a solid wall [3]:

$$z = \left\{c_r u_0'\left[-\frac{1}{(u_k')^2 y_k(1-y_k)} + \frac{u_k''}{(u_k')^3}\ln\frac{y_k}{1-y_k} + \frac{1}{\alpha(1-c_r)^2}\right] + \right.$$
$$\left. + c_i u_0' \frac{u_k''\pi}{(u_k)^3}\right\} + i\left\{c_i u_0' \frac{1+c_r}{(1-c_r)^3\alpha} - c_r u_0' \frac{u_k''\pi}{(u_k')^3}\right\} \tag{3}$$

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 137-139, March-April, 1972. Original article submitted August 30, 1971.

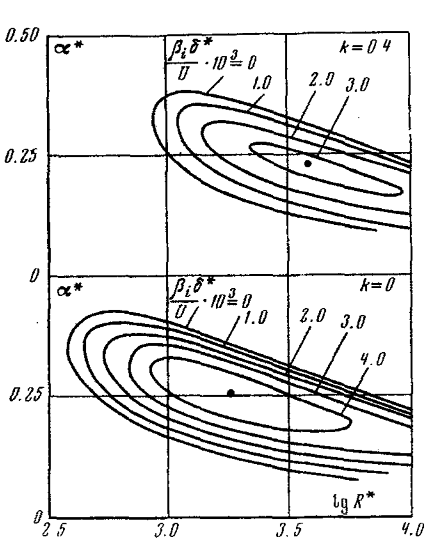

Fig. 1

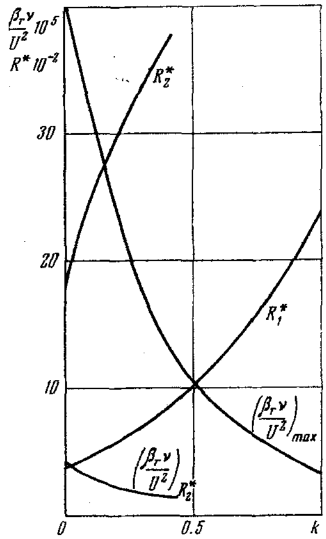

Fig. 2

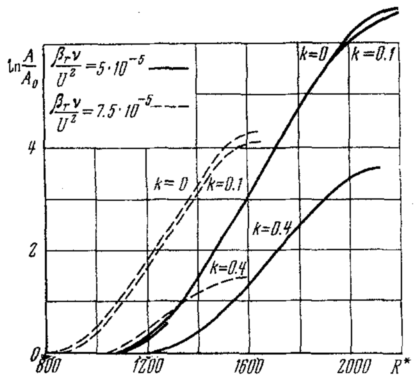

Fig. 3

The left side of Eq. (2) is a universal function, which is similar to the Tietjens function for neutral oscillations, and has been completely tabulated for fixed values of the parameter $c_i/y_k u_k'$ using Hankel functions of the first kind of order $1/3$:

$$F = F_r\left(w, \frac{c_i}{y_k u_k'}\right) + iF_i\left(w, \frac{c_i}{y_k u_k'}\right) \tag{4}$$

$$w = y_k \sqrt[3]{\alpha R u_k'} \tag{5}$$

Equation (2) was solved graphically (using the construction of polar diagrams of its left and right sides) for a Blasius profile for a number of values of $c_i/y_k u_k'$ with zero phase shift $\theta = 0$ and four coefficients of compliance $k = 0, 0.1, 0.4, 1.0$. Calculation results are shown in Fig. 1 in the (α^*, R^*) plane; an asterisk with a symbol corresponds to characteristics determined based on the thickness of the displacement δ^*. With an increase in the ratio $c_i/y_k u_k'$ the area bounded by the curves decreases. The instant of degeneracy of the curve into a point corresponds to the physical impossibility of the further existence of plane perturbations of the type under consideration:

$$\Psi = U\delta f \exp\{i[aX - (\beta_r + i\beta_i)T]\} \tag{6}$$

where Ψ is a dimensional stream function of the perturbing motion, $a = \alpha/\delta$ is the frequency of the perturbation-wave shape, $X = x\delta$ is the longitudinal dimensional coordinate (x is the dimensionless coordinate), $T = t\delta/U$ is the time, t is the dimensionless time and in agreement with the hypothesis of Michel [4] is taken as the beginning of the region of laminar–turbulent transition. The corresponding Reynolds number can be called critical (R_2 or R_2^*), and the minimum Reynolds number on the curve of neutral stability ($c_i = 0$) is the Reynolds number of the loss of stability (R_1 or R_1^*). The dependence of R_1^* and R_2^* on the compliance coefficient k is given in Fig. 2.

There is also a definite interest in the question of the effect of the elasticity of the surface on the growth of velocity pulsations. The longitudinal velocity component of the perturbed motion is found in terms of the stream function (6):

$$v_x = \frac{1}{\delta}\frac{\partial\Psi}{\partial y} A \exp[i(aX - \beta_r T)]$$
$$(A = A_0 \exp(\beta_i T)) \tag{7}$$

where A is the amplitude of the pulsation velocity at the point with coordinate X and corresponding Reynolds number $R_X = UX/\nu$, $A_0 = Uf'$ is the amplitude of the pulsation velocity at the point of the loss of stability

$\beta_i = c_i = 0$ for oscillation of the given dimensionless frequency. This point corresponds to the coordinate X_0 and Reynolds number R_{X0} and R^*_0.

Taking into account that the perturbation wave propagates [5] with group velocity $c_B + \alpha \partial c_B/\partial \alpha$, we can obtain from (7)

$$\ln \frac{A}{A_0} = \int_{T_0}^{T} \beta_i dT = \int_{X_0}^{X} \beta_i \left(c_B + \alpha \frac{\partial c_B}{\partial \alpha}\right)^{-1} dX = \int_{R_{x0}}^{R_x} \frac{\beta_i \delta^*}{R^*} \left(c_B + \frac{\partial c_B}{\partial \alpha}\right)^{-1} dR_x \qquad (8)$$

In the case of a laminar boundary layer on a plane plate we have

$$R^* = 1.72\sqrt{R_x} \quad \text{and}$$

$$\ln \frac{A}{A_0} = 0.668 \int_{R_0^*}^{R^*} \frac{\beta_i \delta^*}{U} \left(c_r + \frac{\partial c_r}{\partial \alpha}\Big|_{R^*}\right)^{-1} dR^* \qquad (9)$$

In Fig. 3 we carry out a comparison of the growth in amplitudes on elastic and rigid surfaces for the dimensionless frequencies $\beta_r \nu / U^2 = 5 \cdot 10^{-5}$ and $7.5 \cdot 10^{-5}$.

A theoretical investigation and calculation results enabled us to establish that with an increase in the value of the compliance coefficient of a streamlined surface there is an increase in the Reynolds number of the loss of stability, the critical Reynolds number, and the length of the pre-transition region (zones located between the points corresponding to R_1 and R_2); there is a decrease in the range of dangerous dimensionless frequencies of the perturbed motion corresponding to the instability zone and also in the dimensionless frequency and the coefficient of growth of the maximum growing perturbations: there is a retardation in the growth of the amplitudes of the pulsation velocities.

LITERATURE CITED

1. A. I. Korotkin, "Stability of a laminar boundary layer in an incompressible liquid on an elastic surface," Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 3 (1966).
2. A. M. Basin, A. I. Korotkin, and L. F. Kozlov, Control of the Boundary Layer of a Ship [in Russian], Sudostroenie, Leningrad (1968).
3. V. V. Droblenkov and A. I. Korotkin, "Investigation of growing perturbations in a laminar boundary layer for various gradients of the external pressure in connection with the problem of the calculation of the transition point," Materials on the Exchange of Experience of the Scientific and Technical Division (NTO) of the A. N. Krylov Institute of the Shipbuilding Industry, No. 127 [in Russian], Sudostroenie, Leningrad (1969).
4. R. Michel, "Determination du point de transition et calcul de la trainee des profils d'aile en liquide incompressible," ONERA Publ. No. 58 (1952).
5. S. F. Shen, "Calculated amplified oscillations in the plane Poiseulle and Blasius flows," J. Aero. Sci., 21, No. 1 (1954).